Annotated Bibliography of Papers Applying Mathematics to Crime

Prepared by students of the Spring 2018 Math Clinic

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References

[1] A.A.Lacey and M.N.Tsardakas. A mathematical model of serious and minor criminal activity. *Euro. Jnl of Applied Mathematics*, 27:403–420, 2016.

> From a great number of models to understand crime, the authors provide a detailed form of the most successful model that describes how the number of criminals evolves in a specific area, which is called diffusion-type differential equations model. In addition, the stochastic variant of the model is examined by the authors, which represents the new kinds of criminals. This paper presents a satisfied agreement by comparing the simulations of model and actual crime data in the Greater Manchester area.The statistical analysis of the data also proves the reliability of the model.

[2] A.C.Kuelling. Statistical correlations of crime with arrests. *Proceedings of Spie*, 12:62–73, 1997.

> This article use a regression analysis to show how crime rate correlates with arrest rate. It gives us definitions of crime rate and arrest rate. The result is that violent crime has little influence on the violent arrest rate, but has a large influence on the property arrest rate.

[3] Sandra Ajimotokin, Alexandra Haskins, and Zach Wade. The effects of unemplyment on crime rates in the u.s. *SmartTech*, 2015.

> This paper utilizes regression models and data acquired from 2013 data statistics to compare the unemployement rate, GDP per capita, high school graduation rates and how many police officers there were per 100,000 inhabitants in correllation to both violet crimes and poverty crimes. The results, as read in this paper, is that there is a positive correlation between unemployment and crime.

[4] Tahani Almanie, Rsha Mirza, and Elizabeth Lor. Crime prediction based on crime types and using spatial and temporal hotspots. *International Journal of Data Mining*, 5:1–19, 2015.

This article reports the results found by applying "Apriory Algorithm", 'Naive Bayesian", and "Decision Tree" methods in two sets of real crime data in Denver, Colorado, and Los Angeles, California. The models found by these methods help predict locations and time most likely to have a specific type of criminal incidents. Aiming for a more accurate way to predict crimes, the study also looks for demographic factors associated with crimes and safety.

[5] M. Andresen. Predicting local crime clusters using (multinomial) logistic regression. *Cityscape: A Journal of Policy Development and Research*, 17:249–261, 2015.

This paper used crime data for Vancouver consisting of calls for service made to the Vancouver Police Department in 2001. The researcher uses multinomial logistic regression since the local crime clusters that emerge from local Moran's I is a dependent category variable. The goal of the article is to predict the hot spot and cool spot of crime.

[6] Jessica J. Asscher, Inge B. Wissink, Maja Dekovi, Peter Prinzie, and Geert Jan J. M. Stams. Delinquent behavior, poor relationship quality with parents, and involvement with deviant peers in delinquent and nondelinquent adolescents: Different processes, informant bias, or both? *International Journal of Offender Therapy and Comparative Criminology*, 58(9):1001–1019, jun 2013.

This article looks at the parent-adolescent relationship quality and association with deviant peers of a group of delinquent and a group of non-delinquent adolescents. It attempts to determine if there is a difference in those two variables between the two groups. Instead, what is discovered is that the perception of those two variables is highly dependent on who is asked, parent or adolescent, and whether the adolescent is a delinquent or not.

[7] B. Berenji, T. Chou, and M. R. D'Orsogna. Recidivism and rehabilitation of criminal offenders: A carrot and stick evolutionary game. *PLOS One*, 9(1), 2014.

This game theoretic model attempts to judge the efficacy of various combinations of rehabilitation and punishment programs against criminal recidivism. In each time step of the model, simulated players have the opportunity to commit a crime or not with probabilities determiened by parameters meant to represent both

rehabilitation programs and punishment programs. The model is designed so that, depending on the parameters set for the punishment and rehabilitation systems, the population will evolve toward a state in which the majority are either reformed and viruous or incorrigble (that is, incapable of being reformed). I think this approach has merit, but the simulations in this paper use arbitrarily defined parameters rather than ones derived from analysis of criminological data, and fails to account for geographic factors. It seems to me, though, some of the game theoretic elements related specifically to recidivism in this model might be used in conjction with other models (perhaps some kind of agent-based simulation of a particular neighborhood) to account for some of the complexity of the decision of whether or not to commit a crime.

[8] H. Berestycki, N. Rodrguez, and L. Ryzhik. Traveling wave solutions in a reaction-diffusion model for criminal activity. *Multiscale Modeling and Simulation*, 11:1097–1126, 2013.

This article studies a reaction-diffusion system of partial differential equations first introduced in [Berestycki and Nadal, J. Appl. Math., 21 (2010), pp. 371399] as a basic model for criminal activity. Under the right assumption that a populations natural tendency towards crime significantly changes the long-time behavior of criminal activity patterns, the authors show that there exists traveling wave solutions connecting zones with no criminal activity and zones with high criminal activity, known as hotspots. Furthermore, this article also studies the problem of preventing invasion of criminal activity by employing a finite number of resources that reduce the payoff for committing a crime in a finite region. By applying theory to the gap problem in the excitable media literature, this article proves existing conjectures in the literature. The article applies partial differential equations of Calculus and Analysis to crime.

[9] D. Birks, M. Townsley, and A. Stewart. Generative explanations of crime: Using simulation to test criminological theory. *Criminology*, 50(1):221–254, 2012.

This paper describes an agent-based model of residential burglaries. Specifically, the authors create a two-dimensional environment populated with potential victims and offenders in which movement is restricted to a transport network. The probability of a crime taking place is a function of the offender's motivation and awareness of a potential target and of the target's attractiveness. Though the paper does not describe the mathematics underlying the model in the level of detail I would require to

be able to modify it to our purposes, it does provide a starting point for thinking about the agent-based approach to modeling patterns in crime. Perhaps the rules governing agent behavior in such a model could be adapted from a game theoretic model? I suspect complex patterns in crime might be effectively explored as emergent properties of actors following relatively simple rules, but I'm not sure this model yet accounts for the decisions made by individual actors.

[10] D. Birks, M. Townsley, and A. Stewart. Generative explanations of crime: using simulation to test criminological theory. *Criminology*, 50:221–254, 2012.

This article presents an ABM (agent-based modeling) of residential bulgary,simulating a world of offenders and potential targets, and examines the impact of environmental criminology mechanisms on the pattern of offending.

[11] Arunesh Sinha Bo An, Milind Tambe. *Improving Homeland Security Decisions*. Cambridge University Press, 2016.

This book applies the Stackelberg game theory model to the problem of security. The authors suggest using the Stackelberg model to determine the most efficient way to allocate limited security resources in order to prevent the most crime. If security schedules are human created, they will be somewhat predictable. This allows criminals to more easily evade law enforcement when they wish to commit crimes. Within the Stackelberg framework, the authors map the potential victim to the leaders role and the potential criminal to the followers role. The dominant strategy is one maximizes the expected utility of the potential victim, given that the potential criminal learns the strategy of the potential victim and adapts. The authors go on to describe several real world scenarios where the Stackelberg model is being used to fight crime.

[12] Hiram Calvo, Salvador Godoy-Caldern, Marco A. Moreno-Armendriz, and Vctor M. Martnez-Hernndez. Forecasting, clustering and patrolling criminal activities. *Intelligent Data Analysis*, 21:697–720, 2017.

Tools that perform pattern recognition analysis of crimes, comprising at the same time forecasting, clustering, and recommendations on real data such as patrolling routes, are not fully integrated; modules are developed separately, and thus, a single workflow providing all the steps necessary to perform this analysis has not been reported. In this paper, we propose forecasting criminal activity in a particular region by using supervised classification; then, to use this information to automatically cluster

and find important hot spots; and finally, to optimize patrolling routes for personnel working in public security. The proposed forecasting model (CR-+) is based on the family of Kora- Logical-Combinatorial algorithms operating on large data volumes from several heterogeneous sources using an inductive learning process. We perform two analyses: punctual prediction and tendency analysis, which show that it is possible to punctually predict one out of four crimes to be perpetrated (crime family, in a specific space and time), and two out of three times the place of crime, despite of the noise of the dataset. The forecasted crimes are then clustered using a density-based clustering algorithm, and finally route patrolling routes were crafted using an ant-colony optimization algorithm. For three different patrolling requirements, we were always able to find optimal routes in shorter time compared to commonly used random walk algorithms. We present a case study based on real crime data from the municipality of Cuautitln Izcalli, in Mexico.

[13] Alejandro Camacho, Hye Rin Lindsay Lee, and Laura M. Smith. Modelling policing strategies for departments with limited resources. *Euorpean Journal of Applied Mathematics*, 27:479–501, 2016.

This paper provides an agent based model for modeling crime patterns, which build on previous research and models on the topic. Provides good overview and references to other work focused on the same topic. This paper builds on previous models by including more activity typically seen in policing practices, such as police responding to non-crime based calls while on duty, and restricting police to certain regions in the model, as they might be assigned to a particular beat in real life. After making such additions, authors concluded that the best policing strategies in regards to resource allocation depend on whether there were no hotspots detected, small hotspots, or large hotspots detected.

[14] Robert Stephen Cantrell, Chris Cosner, and Raúl Manásevich. Global bifurcation of solutions for crime modeling equations. *SIAM Journal on Mathematical Analysis*, 44(3):1340–1358, 2012.

The paper demonstrates the pattern formation in a quasi-linear system of two elliptic equations that were presented as a modal for residential burglary, by Short et al. (Math Models Methods Appl.sci.,18 (2008), pp 1249-1267). The findings show that the average of burglaries of houses that have been burglarized recently is higher than the rate in the large community, which creates hotspots for burglary. The location of these hotspots is determined by the patterns of the model. The formation of spatial patterns is a result of global bifurcation of spatially varying solutions from the spatially constant equilibrium.

[15] B. Cavadas, P. Branco, and S. Pereira. Crime prediction using regression and resources optimization. Technical report, Unversidade do Porto, Portugual, 2015.

This paper presents a "pipe-line" technique that first uses regression to predict violent crime, then uses an Integer Liner Program to optimize the distribution of police officers. The authors use regression to predict violent crime per 100k population, and this appears to be the first time that this approach is used to make predictions about crime. The data is processed using the smoteR algorithm to balance the dataset, then three learning algorithms (SVM, RF, and Multivariate Adaptive Regression Splines) are applied for the prediction task. Next, given the predictions obtained from the regression approach, an Integer Linear Program is formulated to optimize the distribution of police officers across states in the US. The program includes constraints on available number of police officers, individual states budgets, and a minimum number of police officers to guarantee citizen safety. The paper uses real data for violent crime incidents and population numbers, and uses synthetic data for budgets and available police officers. The paper also only discusses 46 states. The work serves as a proof of concept and the authors discuss applying the method to other countries and smaller regions using real data on available capitol.

[16] Andrea L Bertozzi Da Kvang, P. Jeffrey Brantingham. Crime topic modeling. *Crime Science*, 2017.

The goal of this paper is to classify crime into distinct categories while minimizing loss of information. This way, we would be able to better understand what causes crime and what prevents it. These distinct categories are called crime topics, which references the topic modeling methods used to create them.

[17] V. Dabbaghian, V. Spicer, S. K. Singh, P. Borwein, and P. Brantingham. The social impact in a high-risk community: A cellular automata model. *Journal of Computational Science*, 2(3):238–246, 2011.

This paper describes a cellular automata model in which players interact in an environment characterized by high crime rates and high drug use. Specifically, the authors model the interactions between neighboring players in which individuals can either encourage or discourage high-risk behavior such as drug use or crime. I like this model because its approach accounts for infleucnes in social interactions, though it doesn't account for variation in victim populations or in systemic or geographic factors. A possible approach to our problem might include adapting such a model by

adjusting parameters according to crime data and/or inclduing other decision-making factors.

[18] K. de Albuquerque and J. McElroy. Tourism and crime in the caribbean. *Annals of Tourism Research*, 26:968–984, 1999.

This paper comes up with three hypotheses: whether the victimization rates are influenced by tourist density levels; whether tourists are more likely to be victimized by property crime and residents by violent crime, and whether tourists in mass destinations are more likely to be victimized of crime than residents. Then, with longitudinal data from The Royal Barbados Police Force and Caribbean Development Bank, the authors divided crimes into violent crimes and property crimes, and calculated each of the crime rate (per 100,000) by months and categories. The result shows that the last two hypotheses are proved, but it seems there is no seasonal influence in the data. In addition, at the 0.05 confident level, both violent and property crime rate against tourists are negatively related to their flows.

[19] Stephen Demuth and Susan L. Brown. Family structure, family processes, and adolescent delinquency: The significance of parental absence versus parental gender. *Journal of Research in Crime and Delinquency*, 41(1):58–81, 2004.

This article compares the amount of delinquency in single-mother families to single-father families. The purpose being to determine if parental absence or parental gender is the contributing factor to delinquency. The article also considers various family processes that can contribute. Findings include that single-parent families have a higher amount of delinquency than two parent families, and that single-father families have a higher amount of delinquency than single-mother families, though the authors contribute that to family processes.

[20] D.O.Cloninger and R.Marchesini. Crime betas: A portfolio measure of criminal activity. *Social Science Quarterly*, 76:634–647, 2018.

This article want to give us an empirical methodology to measure and evaluate changes in criminal behavior for individual communities. It uses a cofficient, beta, which represents the change in the incidence of a specific crime in a given community to the change of all crime in nation.

[21] Geoffrey H. Donovan and Jeffrey P. Prestemon. The effect of trees on crime in Portland, Oregon. *Environment and Behavior*, 44(1):3–30, 2012.

This article correlates crime rates with trees. Public space trees are compared against total crime, and trees on a homeowners' lot

are compared against burglary and vandalism at that location. Results show that public space trees correlate with lower crime rates in a neighborhood and that large trees at a house correlate with lower chance of burglary and vandalism at that location.

[22] Maria R D'Orsongna, Ryan Kendall, Michael McBride, and Martin B. Short. Criminal defectors lead to the emergence of cooperation in an experimental, adversarial game. *PLOS One*, 2013.

This paper analyzes the effects of 'informants' on cooperation. When considering the prisoner's delimma, informants defect first but cooperate after by punishing other defectors. The goal is to determine if informants are necessary for the emergence of cooperation.

[23] Dale Dzemydiene and Vitalija Rudzkiene. Multiple regression analysis in crime pattern warehouse for decision support.

This article used multiple statistical methods to forecast the main crime tendencies in Lithuania. This article really good because there are numerous graphs that break down the crime. For example, Fig.3 is a graph on the components of crime and the crime statistics. With this graph, at the very top is Crime Statics. Then the tree works its way down to rapes, disordely conduct, and property destruction. Here the graph is exploring all types of crime. Another figure, figure 1. starts with a base. This base looks at crime profile for recent data, crime profile for historical data, and social-economic indicators. The tree then works its way to the top. At the top of the tree, are the preventive measures. Which is in essence this article, looking at the whole picture to arrive at the preventive measures.

[24] G Espejo, G L'HUILLIER, and R Weber. A game-theoretical approach for policing decision support. *European Journal of Applied Mathematics*, 27(3):338–356, 2016.

This paper presents a developed virsion of the stackelberg game for modeling the intraction between police forces and delinquents in police places. the model inspires the development of two games: a classical leader-follower interaction between police and organized criminals and a novel approach between the leader and acting offenders.

[25] J. McDevitt et al. Improving the quality and accuracy of bias crime statistics nationally an assessment of the first ten years of bias crime data collection. Technical Report 218, The Center for Criminal Justice Policy Research Northeastern University, 360 Huntington Ave., Boston, Massachusetts, July 2000.

This paper first separates the process of hate crime reporting to a series of seven steps and compares the number of reporting bias crime to non-reporting hate crime in order to understand what impedes or supports hate crime reporting. To do this, the research groups collected data in different ways: mail survey sent to a stratified probability sample of law enforcement agencies across the country, telephone surveys and other additional interviews with law enforcement representatives. After comparing the number of reporting and non-reporting hate crime, the survey data indicates serious disparities between what officers believed about the prevalence of bias crime and their agencies' official statistics. To improve the reporting accuracy and quality of bias crime, the recommendations focus on building trust between minority groups and local police, improving law enforcement's ability, using supplemental data to 'correct' reported data. This paper really inspires me by its way of solving unreported crime from police stations. Since in my point of view, there are two ways that cause the loss of a report. First is before people realize the category of the crime, the first two steps in a series of seven, and then the misreported or unreported from police station.

[26] David M Ferguson. Alcohol abuse and crime:a fixed-effects regression analysis. 95.

This artilce looked at the juevenilles of New Zealand to see if there were patterns between alcohol abuse and crime. This article used a fixed regression model to model the data. I liked this article because it targeted a certain population ie youth from 14 to 21 Here the main goal was to prove that yes this population can increase the crime rate when dependent upon alcohol abuse. Look at URBAN POVERTY AND JUVENILE CRIME: EVIDENCE FROM A RANDOMIZED HOUSING-MOBILITY EXPERIMENT. This article explores the patterns of crime and youth for the United States. Compare and contrast.

[27] Matthew Fielding and Vincent Jones. Disrupting the optimal forager: predictive risk mapping and domestic burglary reduction in trafford, greater manchester. *Internation Journal of Police Science and Management*, 14:30–41, 2011.

This is a study of the implementation of a predictive model in Trafford, Greater Manchester and its effectiveness in reducing crime rate. Based on previous criminal activity, police issued weekly reports to their patrolling units with maps of hotspots, which included a temporal graph with days and times where criminal activity was at a higher risk. The police were instructed to

use this data to inform their patrolling routes, and in particular to be present in higher risk areas at the right time to act as a 'capable guardian' who could disrupt the anticipate criminal patterns. Over the year-long study, the police department saw a 26.6 percent decrease in domestic burglaries, with targeted patrol areas showing the greatest decrease. This is a promising study in the potential impact of data-driven decisions as far as resource allocation.

[28] Matthew Gerber. Predicting crime using twitter and kernel density estimation. *Decision Support Systems*, 61:115–125, 2014.

This article presents research about predicting crimes when adding information obtained from Twitter into predictive models using kernel density estimation (KDE). In addition to reviewing the KDE approach on predicting crimes, the article also describes how to obtain data from Twitter, and how to implement it into the density function. The paper compares the standard KDE model and the Twitter-based model. The paper also mentions the usage of MALLET package and its computational efficiency.

[29] G.Espejo, G.L'huillier, and R.Weber. A game-theoretical approach for policing decision support. *Euro. Journal of Applied Mathematics*, 27:338–356, 2016.

Recently, people pay more attention on the quantitive methods to solve the security-related problems. In this paper, they proposed a game-theoretical approach to get a model and find the interaction between the police force and the offenders in public places. Also, in the application, they expand the traditional model so that they made one police face with many offenders simultaneously instead of the classic one-to-one relationship because of the limited police resources. Moreover, in these offenders, they may be organized or they acted selfishly. During the complicated situations, if we want to find the effect of crime displacement under police surveillance, the results using data from the model can preform better than traditional strategies. First, they proposed the security problems and show that we need the new model.Then, they introduced some concepts. Next, the models for crime prevention developed in this work are represented and we can then applicate them. In the last, concludes this paper and seek for a research direction.

[30] Y. Gu, Q. Wang, and G. Yi. "stationary patterns and their selection mechanism of urban crime models with heterogeneous near-repeat victimization effect". *"Journal of Applied Mathematics Cambridge"*, "28":"141–178", "April 2016".

" Key words in the article include Urban Crime Model, Nonlinear diffusion, Pattern Formation, and Stability Analysis. The three main parts that make up the crime patters are social disorganization, subculture, conflict theories. There are models the author used to create a pattern for repeated crime where there are functions of space time and an equation that captures the attractiveness of houses. There is also criminal population density function. Crime hot spots are the clustering of crime data in urban resident burglary in certain locations. Some neighborhoods have a higher rate of burglaries than others. There is quite a bit of detail that goes into the variables and conditions in the different equations, which I will not go into detail here. Overall, the article is educational, but covers quite a vast amount of information that would take years to understand and form models to fit these situations."

[31] Yu Gu, Qi Wang, and Guangzeng Yi. Stationary patterns and their selection mechanism of urban crime models with heterogeneous. *European Journal of Applied Mathematics*, 28:141–178, 2017.

This paper expands on the work of Short et. al. (2008), who proposed a model of urban crime. The authors of this paper use this model as a base, while also considering the near-repeat victimization effect. Repeat victimization refers to a situation where a previous victim of a crime becomes a victim again. Near-repeat victimization on the other hand may not involve the exact same victim, but involves victimization of individuals very similar to the previous victim in personal characteristics, behaviors, or situational factors. This article would be appropriate for any group focused on modeling with partial differential equations, as this is the main mathematical tool used in this paper.

[32] Yu Gu, Qi Wang, and Guangzeng Yi. Stationary patterns and their selection mechanism of urban crime models with heterogeneous near-repeat victimization effect. *European Journal of Applied Mathematics*, 28(1):141–178, 2017.

This paper discusses the modifications that should be made to the two PDE's that generalize the urban crime model proposed by Short et al. (2008 Math. Model Methods Appl. sci 18, 1249-1267). It investigate pattern formations in that reaction adevection-diffusion systems with non-linear diffusion over multidimensional bounded domains subject to homogeneous Neumann boundary conditions. The resulting solutions can model the well-observed phenomenonof aggrgation in urban criminal activities.

[33] E. Guttel and B. Medina. Less crime, more (vulnerable) victims: Game theory and the distributional effects of criminal sanctions. *Review of Law and Economics*, 3:407–435, 2007.

This paper examines the effects of sanctions on the distribution of victims across sectors of potential targets (for example, comparing the distribution of targets who can defend themselves versus targets who can not). This paper also uses insights from game theory to show that criminal sanctions impact overall crime rates. This paper shows that the level of sanctions affects the behavior of the criminal and the police differently. The offender utilizes the magnitude of the sanction to determine the volume of his activity; however, sanctions do not change the offender's efforts across different types of victims. The paper demonstrates that harsher punishments serve the less vulnerable targets when enforcement efforts are effective. Another observation of this study is that the police and the offenders both apply mixed strategies to achieve a Nash Equilibrium. Success in this study was measured by minimizing crime and maximizing apprehension. The results of this study suggest that police locations should be randomized across neighborhoods, which differs from other models that suggest police should be stationed at hot spots. Although the study is over ten years old, it's results are still relevant for providing alternative strategies to law enforecement. This article would be useful to anyone who wants to study alternative methods of crime reduction from a game theory perspective.

[34] Cynthia C Harper and Sara S McLanahan. Father absence and youth incarceration. *Journal of Research on Adolescence*, 14(3):369–397, sep 2004.

This article measured the likelihood of incarceration among adolescent males from father-absent homes. It finds that there are many other factors, such as teen parenthood, low parent education, and poverty that increase the likelihood of incarceration, but even with those factors accounted for, there was still a significant increase in likelihood of incarceration in father-absent homes. The article found the likelihood is greatest in stepfather homes.

[35] Health and Wellness Resource Center. Fighting violent gang crime with math. *NewsRx health and science*, 15:104, 2011.

A UCLA program. This mathematicians analysis more than 1000 gang crimes and suspected gang crimes which occurred more than 10 years period in, an East Los Angeles police district, Hollenbeck. This algorithm explored gang activity patterns to produce the best probability of which gang, or whcih three gang, may

have the chance for the crimes.(interesting topic and do not have detailed article).

[36] J. R. Hipp. General theory of spatial crime patterns. *Criminology*, 54:653–679, 2016.

The author proposed a general theory for examining the spatial distribution of crime. More specifically, the author estimates the spatial distribution of offenders, targets and guardians by modeling their respective expected movement patterns across space and time.

[37] Shital Jayantilal, Silvia Ferreira Jorge, and Ana Ferreira. Portuguese anti-money laundering policy: a game theory approach. *Eur J Crim Policy Res*, 23:559–574, 2017.

The article talked about how to use the game theory approach to study the efficiency of the combat against money laundering in Portugal.Also, the paper studies how the increasing sanctions effect the combat. The result shows the increasing sanctions tend to augment the efficiency of the combat against money laundering in Portugal. The article first introduce the money laundering problem and some relevant cases in the Portugal. Then, they present the model and discuss the results. Finally, they conclude the influencing and limitation of the efficiency of the Portugal's anti-money laundering policies and talked about the research direction.

[38] John R. Hipp (john.hipp@UCI.edu). General theory of spatial crime patterns. 2016.

This article is analyzing theory of spatial distribution of crimes by estimating the the unique distribution of offenders and their targets along with their expected movement patterns depending on the location and different timing. The model that the author uses combines information on the locations of offenders with their spatial movement patterns, including their location to have a better estimation about the crime location. Thus, the equations that is been used in this article provide the ideas that involved in theory explicit and highlight the locations where the offenders are hacking and when. Then, by creating a model that will help to highlight the locations of offenders and their targets, this theory will then provide an accurate study of crime and affects in those areas with higher crime patterns. Also, the equations that are provided in this article will help to locate the travel distance for offenders to spot their movements as they travel from one location to another. Furthermore, those equations could be used in order to simulate the crime patterns and the location changes

and so they provide a study for various places and crimes where they happen frequently.

[39] Shane D. Johnson. a brief history of the analysis of crime concentration. *Euro. Jnl of Applied Mathematics*, 21:340–370, 2010.

In this article of reviewing some of available evidence and providing illustrations of the types of analysis of crime concentration, Johnson focuses on spatial and spatio-temporal analysis. For the spatial concentration, the author demonstrates concentration at levels of area, street and point, respectively. For the level of point, the author also pays attention to the repeat victimization. Given that the previous studies show that crime is concentrated at a range of spatial scale, recent researches points out that crime clusters in space and time with a regularity might contribute to a better prediction of crime. Due to the limitation of application of formal Mathematics implicated by a few exceptions, the author provides a dominating aim of animating interest in the field.

[40] T. Kolokolnikov, M. J. Ward, and J. Wei. The stability of steady-state hot-spot patterns for a reaction-diffusion model of urban crime. *Discrete and Continuous Dynamical Systems - B*, 19:1373–1410, 2014.

This article examines the existence and stability of the hot-spot of criminal activities in the reaction-diffusion model. Non-local eigenvalue problem, Hopf Bifurcation, etc are being used as methodologies.

[41] A. Lacey and M. Tsardakas. "a mathematical model of serious and minor criminal activity". *"Journal of Applied Mathematics Cambridge"*, "27":"403–421", "Febuary 2016".

" The article uses data from an area of criminal activity in Manchester. The models used in this study are diffusion, differential equations that help predict and show the history of crime in a certain area. The article focuses in on different types of crime including serious crime and minor crime. The article describes a theory that involves a correlation between the rate of change of a minor crime to a serious crime. The article also uses a stochastic model in order to give more flexibility of the model itself. I will do more research on a stochastic model, but from my knowldege it is a model that allows us to find the probability of a certain outcome in regards to certain random variables. Overall, this is a very interesting article that has more visual aids and data than the other two that I looked at. I also think it is easier to understand and more of a task that I could grasp doing."

[42] A. A. Lacey and M. N. Tsardakas (mtsardakas@gmail.com). Mathematical model of serious and minor criminal activity. *European Journal of Applied Mathematics*, 27:403–421, 2016.

The author of this article have been using differential equations and different mathematical methods to solve and understand crimes patterns. The researchers have shown that one of the best ways to solve crimes is by using model equations that will help to describe the crime scene and so this will help to solve the crimes or to reduce the chances of having them on the study areas. Researchers have found different resources to understand the way the crimes could occur and the causes behind them, and that partly was done by using the differential equations. This is because crimes are part of the society, so understanding the reasons behind crimes will help to eliminate the crimes and to fight against offenders. There are some specific amount of research that has been conducted on the spatial and temporal distribution of crimes in this article. Thus, the author has collected the different behaviors in the hotspots that he drew his focus on, then he derive a continuous model that captures those behaviors. The purpose of this article is to define the relation between serious and minor crime in hotspot areas, then to define a quantitative mathematical model that help to capture the distribution of the serious and minor crimes in the same areas

[43] A.A. Lacey and M.N. Tsardakas. A mathematical model of serious and minor criminal activity. *Journal of Applied Mathematics*, 27:403–421, 2016.

Lacey and Tsardakas explore a descriptive model for major and minor crime. A diffusion-type differential equations model that implements both deterministic and stochastic methods to fit real crime data from the Greater Manchester area. The paper does not aim to reduce crime directly, but to study the relationships that are indicative of crime. The scholastic diffusion-type differential equations model is effective at fitting the Greater Manchester crime data and holds much promise if explored further.

[44] Andrew Alfred Lacey and MN Tsardakas. A mathematical model of serious and minor criminal activity. *European Journal of Applied Mathematics*, 27(3):403–421, 2016.

This paper offers a more detailed form of a diffusion-type differential equations model which estimates the number of criminals in a certain area. This form explores a stochastic variant that represents the generation of new criminals. Finally solutions from both models is compared with actual data for the Greater Manchester area.

[45] Andrew Alfred Lacey and MN Tsardakas. A mathematical model of serious and minor criminal activity. *European Journal of Applied Mathematics*, 27(3):403–421, 2016.

In this article, the authors use a data-analysis viewpoint to explore the relation between serious and minor crime and set a quantitative mathematical model that captures the dynamics that rule the temporal distribution of serious and minor crime. This article is a good resource for future work.

[46] Jane Law, Matthew Quick, and Ping Chan. Bayesian spatio-temporal modeling for analysing local patterns of crime over time at the small-area level. *Journal of Quantitative Criminology*, 30:57–78, 2014.

This article, placed and constructed in Ontario, Canada, employs the first Bayesian Spatio-Temporal model for crime trend anaylsis on a large scale map. They indentified hot spots and cold spots in areas and observed differential trends in property crime. Overall, the researchers found this model useful for analyzing risks and trends in crime.

[47] Nicola Lettieri, Delfina Malandrino, and Luca Vicidomini. By investigation, i mean computation a framework to investigate the societal dimension of crime. *Trends in Organized Crime*, 20:31–54, 2017.

This article, written by students in Italy presents a holistic approach to criminal analysis. They argue that there may be various advanced technology out there to aid in criminal analysis or criminal investigation but it is not the easiest to use or most readily understood by the people who need it (i.e. police officers, prosecuters, etc). Also, much statistical criminal analysis may not be easily connected to understanding the trends or criminal behavior. The authors strive to connect computational methods with the social constructs of crime to better understand and handle criminals in the age of technology.

[48] S. Levitt. The relationship between crime reporting and police: Implications for the use of uniform crime reports. *Journal of Quantitative Criminology*, 14:61–81, 1998.

This paper addresses whether the size of a police force would systematically affect the willingness of victims to report crimes or a police department's propensity to officially record victim crime reports. According to the data, victims may be more likely to report crimes to the police when the likelihood of a crime being solved is high. Additionally, the availability of a police officer at the scene of a crime may also lead to more crime reports. The three data sets investigated in the paper provided roughly the

same magnitude of reporting bias. The paper also suggests that the number of crimes reported increased as a function of the number of sworn officers per capita. Taking reporting bias into account makes the hiring of additional police beneficial from a cost-benefit perspective, but it does not explain the inability of past studies to uncover a negative relationship between the size of the police force and crime rates. Although this paper is twenty years old, the hypothesis proposed is still current and valid; the negative relationship between size of police force and crime rates is currently frequently observed.

[49] S. D. Levitt. The relationship between crime reporting and police: Implications for the use of uniform crime reports. *Journal of Quantitative Criminology*, 14:61–81, 1998.

This paper investigates three data sets in four ways to measure reporting bias. Even if each of datasets has shortcomings, the final result benefits a lot from their advantages and different features in data structures. The author uses cross-sectional variation from the NCVS Cities Surveys (which contains the cities information), annual NCVS over the period 1973-1991 (without cities) in his first two approaches. After assuming that murders are always reported to the police and will be immune to reporting bias, the author expect to see an increase in the ratio of other crimes to murders as the number of police per capita also increases. The last method investigates not only changes in reporting behaviors, but also the effect of police recording practices on reporting rates. It inspires me by its idea in measuring the effect of police power (not just set police station as a dummy variable) on crime rate, and it's intelligent way in using the ratio to check bias. The conclusion is the likelihood that a crime will be reported is positively correlated to the number of officers per capita.

[50] Steven D. Levitt. The relationship between crime reporting and police: Implications for the use of uniform crime reports. *Journal of Quantitative Criminology*, 14:61–81, 1998.

This journal article attempts to show the presence of reporting bias in crime reports by examining the relationship between the size of a police force and reported crime. The author uses three approaches, two of which use victimization data and estimations of the size of the police force (because no direct measure is available) and employs a weighted least squares technique to determine the "true" rate of crime occurrence compared to the recorded values and extracting a specific coefficient to capture reporting bias. The author himself comments that the data used for these estimates is out of date for the time of the study. But

the paper employs an interesting theoretical method in the third approach. The author argues that murder is virtually always reported, and therefore immune to reporting bias. He compares the reporting of murder to non-murderous crimes to determine rates of reporting bias present in the dataset and compares this information to the size of the police force. The findings of the third approach are clearer than the first two approaches, but still do not draw any significant conclusions to support the authors initial claim. Much of this paper (the data and methods) are grossly outdated, but the theoretical approach of identifying reporting bias by comparing rates of murder to non-murder crimes, is an interesting method that could be applied in current research.

[51] Xiannuan Liang and Yang Xiao. Game theory for network security. *Ieee Communications Surveys and Tutorials*, 15:472–487, 2013.

The article talked about the security issues of the networks today and how to use game theoretic approaches as a useful tool to handle the network attacks. It first introduced people some useful definitions and classification of the game. The paper also review the existing solutions based on game theory for network security problems and classified applications and modeling of the game methods. Besides, it also talked about the limitation of the these game theory. Finally, it pointed out the research direction in the future.

[52] M. Lin. More police, less crime: Evidence from us state data. *International Review of Law and Economics*, 29:73–80, 2009.

This paper used panel data of 51 US states covering the period from 1970 to 2000. The researcher sets state tax rate as an instrumental variable for local police numbers and use Two-stage least square (2SLS) method to conclude that the elasticity of police presence with respect to crime is about 1.1 for violent crime and 0.9 for property crime.

[53] Hua Liu and Donald Brown. Criminal incident prediction using a point-pattern based density model. *International Journal of Forecasting*, 19:603–622, 2003.

The article introduces an extension of a crime clustering method using kernel density estimation by adding offenders' preferences in crime site selection. The method is made based on the belief that a future crime incident in a location is linked with the type of crimes which have happened in the past and other independent spatial features. The article describes in detail the procedure and the math behind the transition density model.The density estimates are converted into percentile scores, and the higher a score

is, the more likely that an incident will happen at a specific location. The paper suggests three versions of the methods: GMM, WPK, and FPK. They are differentiated by the way to estimate the transition density model.

[54] D. Lloyd, N. SANTITISSADEEKORN, and M. Short. "exploring data assimilation and forecasting issues for an urban crime model". *"Journal of Applied Mathematics Cambridge"*, "27":"451–478", "October 2015".

" Contributing factors that helped predict crime spikes included attractors, self-exciting effects of crime. This information helps tell the amount of copy cats or others who repeat crime for self-excitement. With the time and place seeming random you can use those factors in a probability density equation adding in self-excitement to predict crime spikes before they happen. The crime rate density at location x at time t is shared as a linear superposition of the background rate. Which represents the crime rate. There are also location figures at certain times. They show the self-excitement and crime history that can be used to predict future time and location for future crimes. In some cases you will add crime history locations and times for a more complex problem that will predict the location of time of future problems with more accuracy."

[55] M. McBride, R. Kendall, M.R. D'orsogna, and M. B. Short. Crime, punishment, and evolution in an adversarial game. *European Journal of Applied Mathematics*, 27:317–337, 2016.

This paper examines the theoretic properties of the model introduced in [5]. Since converting citizens to informants is not sufficient enough to guarantee the best response, alternative methods are used to help the system reach cooperation. This paper examines the use of moderate punishments for criminals and alternative strategies that do not reliably convert citizens to informants. Although initial results indicate dystopia may be achieved when no informants were present, a small deviation from this would cause the system to switch to semi-utopia. Updated results from this paper suggest the severity of the punishment assessed on criminals has significant impact on whether the systems went to dystopia, utopia, or semi-utopia. The punishments must not be too severe since it possible for criminals to transition to villains instead of informants. This recent paper offers valuable insight in to the effects of punishment on the cooperation of criminals. This article is useful to anyone who wants to analyze crime and punishment from a game theory perspective.

[56] Michael McBride, Ryan Kendall, Maria R D'orsogna, and Martin Short. Crime, punishment, and evolution in an adversarial game. *Euorpean Journal of Applied Mathematics*, 27:317–337, 2016.

This paper builds on [4] and suggests alternative strategies that arent reliant on converting citizens to informants. One key point of the paper is that in adversarial games there can be multiple equilibria, of which one is efficient, as opposed to social dilemma games where there is one equilibrium that is inefficient (such as Prisoners Dilema). In the initial paper, dystopia could be achieved when no informants were involved, but small perturbations of this would at least cause the system to evolve to semi-utopia. In the new paper, the punishment of criminals had a large impact on whether the systems went to dystopia, utopia, or semi-utopia, with the possible implications towards policy being that punishment for criminals must not be so harsh that it causes the transition of informants to villains.

[57] B. Mesquita and L. E. Cohen. Self-interest, equity, and crime control: A game-theoretic analysis of criminal decision making. *Criminology*, 33:487–518, 1995.

This paper utilizes the logic of game theory to solve a crime game to determine which social situations are likely to lead someone contemplating criminal activity to engage in lawfully conforming behavior instead. In many situations, the outcome depends on the actions/characteristics of both participants in the game, not just the criminal. This model assesses the decision to engage in a crime or to abide by legally acceptable norms of behaviour that include the notion of fairness and equity. Race and gender affects each individuals perception of what acceptable norms of behavior are. The analysis from this study suggests that the behavior of criminals vary even if they commit the same crime, increasing expenditure to apprehend criminals reduces crime, increasing the severity of the punishment weakly reduces crime, increasing trust in the government reduces crime, for an expected level of trust in the government, poor people commited more crimes than wealthier people, and crime reduces more if government provides a mix of increased chance of apprehension and increased long-term rewards through public policy. This article is useful to anyone who wants to analyze crime from a game theory perspective or would like to mathematically analyze the psyche of a criminal.

[58] G. O. Mohler, M. B. Short, P. J. Brantingham, F. P. Schoenberg, and G. E. Tita. Self-exciting point process modeling of crime. *Journal of the American Statistical Association*, 106:100–108, 2011.

The article proposes that spacetime clustering models by self exciting point processes are well suited for criminological applications. Based on residential burglary data provided by the Los Angeles Police Department, the authors use a fully nonparametric estimation methodology to discern the form of the spacetime triggering function and temporal trends in the background rate of burglary. The goal is to illustrate the implementation of self-exciting point process models in the context of urban crime. The authors also make crime forecast through comparing their methods and hotspot maps, and then discuss the future usage of their methods in criminology and other fields. The article applies the self exciting point processes of the Probability and Statistics to crime.

[59] G. O. Mohler, M. B. Short, Sean Malinowski, Mark Johnson, G. E. Tita, Andrea L. Bertozzi, and P. J. Brantingham. Randomized controlled field trials of predictive policing. *Journal of the American Statistical Association*, 110:512:1399–1412, 2016.

Epidemic-Type Aftershock Sequence Model for Crime Prediction, in other words cloud-based, machine learning, predictive modeling was implemented by the Los Angles Police Department and the Kent Police Department to identify 150 X 150 meter crime hot-spots. The predictive model is fed every hour, and updates hot-spots each morning. The model considers crime history, and focuses on aftershock crimes, or crimes that happen with a high likelihood after other crimes or local events. The model was designed to detect a contagious sequence of events leading to a crime before the crime occurs, than would dispatch police officers to patrol the hot-spot to deter the action from occurring. As a result, treatment hotspots exhibited 23 percent fewer crimes than controls.

[60] M.Tayebi, UWE. Gl/"asser, M.Ester, and P.Brantingham. Personalized crime location prediction. *European Journal of Applied Mathematics*, 27:442–450, 2016.

This article presents a spatial analysis model called "Crime Tracer". The goal of the model is to predict locations in a coldspot in urban areas where known offenders are likely to perform crimes. The model is made based on a criminal theory that offenders are likely to commit crimes at locations they are familiar with. Thus three main components are taken into account in the model: the offenders' criminal history, the road network and the co-offender network. From these three components, a system of "nodes" and "paths" in an "activity space" of offenders are created. Using random walk method, the probabilities that

crimes could happen at each potential location are calculated. The model was evaluated via three criteria: precision, recall, and utility. Recall computes the proportion of correct criminal location predictions over all criminal locations.Precision computes the proportion of correct criminal location predictions over all predictions. Utility computes the percentage of offenders with at least one correctly predicted crime location. The model is tested by using data from police records for crimes in Metro Vancouver.

[61] Peter Phillips and Ickjai Lee. Mining co-distribution patterns for large crime datasets. *Expert Systems with Applications*, 39:11556–11563, 2012.

This article presents an analysis technique that can recognize patterns of geospatial crime large data. The same pattern of different sets gives features that might be helpful and important in predicting and preventing crime incidents. In the article, the authors model aggregated data as graphs which store crime distributions within a given region. Two techniques to aggregating data and pruning the graphs: one using a global minimum edge weight, and one using the minimum spanning tree. After pruning the graphs, they explore the graphs with the same geospatial distribution change. The result patterns depict datasets with similar distribution in specific spatial regions. The article shows the results on simulated data sets and on real datasets from suburbs of Brisbane, Australia.

[62] Gruenewald PJ, Ponicki WR, Remer LG, Waller LA, Zhu L, and Gorman DM. Mapping the spread of methamphetamine abuse in california from 1995 to 2008. *American journal of public health*, 2013.

Methampetamine abuse and dependence at hospitals in California. This was done by geocoding methamphetamine discharges to residential zip codes from 1995 through 2008. This was modeled using a Bayesian Poisson model. In addition to the math, this article utilized graphs to demonstrate methamphetamine abuse and dependency. For example, figure 5 showed a map of income, pop. density, household size, race/ethnicity and how it relates to meth abuse/dependency. Lastly, I liked this article because it utilized zip codes to try and target the hotspots.

[63] R. Pradiptyo. Does punishment matter? a refinement of the inspection game. *Review of Law and Economics*, 3:197–219, 2007.

This article aims to refine the inspection game Tsebelis proposed (1989, 1990, 1993; BOT, 1990) by disaggregating the game payoffs and then using findings from empirical studies to reconstruct the game. Based on assuming that the severity of punishment

has a positive correlation with the costs of delivering court sentences, the author argues the comparability between Beckers and Tsebelis models. The result the author found is that severity of punishment increase will reduce the likelihood of enforcement of the law. And he also discusses the impact of crime prevention initiatives on individuals offending behavior. The article applies the inspection game of the game theory to crime.

[64] David Seliger. Visualizing criminal networks to help police solve crime. *Corr77*, 2012.

This article delves into the use of link analysis to see the connections between people and how they may one day commit a crime. The study utilizes facebook and creates graph, seperating friends and family and then aquaintences to see how the crime network and the social network overlap.

[65] Jarrod S. Shingleton. *Crime Trend Prediction Using Regression Models for Salinas, California.* PhD thesis, Naval Postgraduate School, Monterey, California, June 2012.

Binomial regression, ordinary least squares and Poisson regress were used to identified six contributing factors to crime. The factors: police department budget, local correctional facility overcrowding, unemployment rate, number of police officers, number of vacant houses and intercept rate, were show to have a high correlation with violent crime, homicide, and assault.

[66] M. B. Short, A. L. Bertozzi, and P. J. Brantingham. Nonlinear patterns in urban crime: Hotspots, bifurcations, and suppression. *SIAM J. AppLIED Dynamical Systtem*, 9:462–483, 2010.

This research is based on a linearly model and aim to develop a weakly nonlinear analysis and amplitude equations for the model to address that the homogeneous system can be unstatble to disturbance of specific wavenumbers under certain parameter regimes. The model consisted two coupled including nonlinear PDEs that may describe the formation and dynamics of crime ?hotspots? –spatio-temporal clusters of high crime which was in both the one-dimensional and two-dimensional. In the end, the researchers also compared supercritical spots and subcritical hotspots with the success of suppression.

[67] M. B. Short, P. J. Brantingham, and M. R. D'orsogna. Cooperation and punishment in an adversarial game: How defectors pave the way to a peaceful society. *Physical Review Journal*, 82, 2010.

This paper uses a game theory model to explain criminal behavior. The paper considers an idealized society where possible

strategies compared to criminal behavior are divided into four categories across two distinct domains: those who either will or will not commit crimes and those who either will or will not report the crime. The model utilizes a Poisson process for the amount of time until the next crime, and selects a criminal and then a victim from the remaining population. After the outcome has been established, players may switch to a different type of citizen. In this model, reduction of crime is noticed when their is a large proportion of informants (players who will commit a crime and work with authorities). This is an excellent introductory paper on the game theory model and should be utilized by anyone unfamiliar with two player games. The results of this paper are still valid as it is frequently referenced in other game theory studies.

[68] M. B. Short, P. J. Brantingham, and M. R. D'orsogna. Cooperation and punishment in an adversarial game: How defectors pave the way to a peaceful society. *Physical Review Journal*, 82, 2010.

Game theory model of crime (though model is not necessarily restricted to crime) in which a fixed population has people who commit crimes and those who dont, and people who report crimes and those who dont (thus creating 4 unique types of citizen). The model follows a Poisson process for the time of the next crime, and selects one of two types of criminals (those who would cooperate and those who wouldnt) and 1 victim from the remaining population. Depending on the outcome of each crime, as well as the possible punishments that follow, players may switch strategies. In the end, the key to crime reduction rests in the proportion of informants, people who may commit a crime, but who also are willing to work with authorities.

[69] M. B. Short, M.R. D'Orsogna, P.J. Brantingham, and G.E. Tita. Measuring and modeling repeat and near-repeat burglary effects. *Journal of Quantitative Criminology*, 25:325–339, 2009.

There is great importance in understanding trends and probabilities of repeat offenses in crime. Researchers in Los Angeles use counting principles to find the probability of time intervals between offenses.

[70] Marth B. Short, P. Jeffrey Brantingham, Andrea L. Bertozzi, , and George E. Tita. Semismooth and semiconvex functions in constrained optimization. *Dissipation and displacement of hotspots in reaction-diffusion models of crime*, 107:3961–3965, 2010.

According to the empirical evidence for how offenders? movement and meet with potential victims or targets, the researchers

build a program using reaction-diffusion partial differential equations to analyze the dynamics of crime hotspots including supercritical and subcritical hotspots. This analysis also gave a mechanistic explanation for the failures of observe crime displacement in experimental field.

[71] M.B. Short. The math behind the scene of the crime. *Physics Today*, 2:58–59, 2014.

This article talks about how to use a math model to illuminates why crime hot spots form, and it give police a guide to solve this problem. The case is from a region of Californis's San Fernando Vally. It uses the knowledge of linear combination which has two factor: An intrinsic attractiveness and a dynamic attactiveness. Author also enumerated some mathematical fomulas in this article.

[72] M.B. Short, M.R D'orsogna, P.J.Brantingham, and G.E. Tita. Measuring and modeling repeat and near-repeat burglary effects. *Journal of Quantitative Criminology*, 25:325–339, 2009.

This paper is focused on modeling exact-repeat burglaries, where the event occurs at the exact same location, as well as near-repeat burglaries taking place within a set neighborhood of the original burglary. Analysis looked at burglary data in single family homes in Long Beach, CA from 2000-2005. Paper presents two models, one which is based on random events based on risk of individual homes, while the other takes into account event dependence (same houses being struck multiple times). The paper concludes that the event dependence is a better model. Model one is called a Random Event Hypothesis and assumes that burglaries are a stochastic process and can be modeled as a Poisson process with parameter lambda. The first analysis of this uses a moving window method of measuring time between burglaries, which supports the REH model. However, using a fixed window method contradicts the REH model and suggest some event dependence. The final model which uses a weighted probability distribution for each home depending on its state (determined by recent break-in data) produces the most robust model and implies that at least some of the data is event dependent.

[73] Milind Tambe, Manish Jain, James Pita, and Albert Jiang. Game theory for security: Key algorithmic principles, deployed systems, lessons learned. *IEEE*, 2013.

The goal of this paper is to develop an algorithm to design security schedules. Since we have limited resources for security,

schedules must be made while taking different priorities int account. Here, computational game theory is used to created such schedules.

[74] Mohammad A. Tayeb, Uwe Glässer, Martin Ester, and Patricia L. Brantingham. Personalized crime location prediction. *European Journal of Applied Mathematics*, 27, 2016.

This article is focusing on using the different strategies and other techniques to reduce the crime and prevent them. One of the strategies it uses is to illustrate a walk-base model by using a crime-tracer to predict the location of the areas with high crime intensity away from the hotspots areas. Based on the research that was done by the author, it appears that there is a high chance of crimes to happen outside the hotspots and that's because of the high attention that was drawn on the hotspot areas. It uses the law enforcement to face inevitable the high rate of urban crime that could be one of the main reasons to affect the growth of urban population. Based on the crime pattern theory, it was conclude that offenders usually select targets in areas that they most familiar with, hotspot areas could be included in this target but not necessary to be part of them. Throughout this study, it has been shown that the urban crimes does not happen constantly, instead, there are some urban areas that have higher rate of crimes, which what effect on the population growth. Due to these results, the author has shown that there are a lot of attention was drawn more on spatial crime analysis such as crime hotspots and areas with an excessive amount of spatial crime analysis. Then, this article is providing a crime model data in different properties of interest in the analysis of criminal networks and their locations based in crime intensity. Thus, by using the right algorithms and the mathematical equations as needed, the crime data analysis methods will provide an accurate information and data about the spatial decision makers of criminal individual offenders and their social activity along with their behavioral patterns. Therefore, by providing the ideas of this article, researchers can use these methods of application for criminal investigations to help limiting the crimes.

[75] Mohammad A Tayebi and Uwe Glässer. Personalized crime location prediction. In *Social Network Analysis in Predictive Policing*, pages 99–126. Springer, 2016.

This paper introduces a personalized random walk-based approach to spatial crime analysis and crime location prediction outside of hotspots. Spatial behaviour of known ofenders is represented by a probabilistic model with in their activity spaces.

Crime pattern theory shows that offenders choose their victims near most familiar places.

[76] Mohammad A. Tayebi, Uwe Glässer, Martin Ester, and Patricia L.Brantingham. Personalized crime location prediction. *Euro. Jnl of Applied Mathematics*, 27:422–450, 2016.

Given that crime hotspots, where the crime density or rate is significantly high, has drawn plethora attention of researchers, the authors present CRIMETEACER, a personalized random walk-based approach to spatial crime analysis and crime location prediction outside of hotspots. Theoretically, the authors cite Crime Pattern Theory, which illustrates that criminals prefer to perform crime at their familiar location. They derive a probabilistic model of spatial behavior of known offenders with their activity spaces from the theory. Experiments in the last part of the article is designed base on a large crime dataset in reality. It shows that CRIMETRACER is better than all other methods used for location recommendation evaluate by the authors.

[77] WH Tse and MJ Ward. Hotspot formation and dynamics for a continuum model of urban crime. *European Journal of Applied Mathematics*, 27:583–624, 2015.

The existence, stability, and dynamics of localized patterns of criminal activity are studied for the reactiondiffusion model of urban crime introduced by Short et al. (Math. Models. Meth. Appl. Sci. 18(Suppl.), (2008), 12491267). In the singularly perturbed limit of small diffusivity ratio, this model admits hotspot patterns, where criminal activity of high amplitude is localized within certain narrow spatial regions. By using a combination of asymptotic analysis and numerical path-following methods, hotspot equilibria are constructed on a finite 1-D domain and their bifurcation properties analysed as the diffusivity of criminals is varied. It is shown, both analytically and numerically, that new hotspots of criminal activity can be nucleated in low-crime regions with inconspicuous crime activity gradient when the spatial extent of these regions exceeds a critical threshold. These nucleations are referred to as peak insertion events, and for the steady-state problem, they occur near a saddle-node bifurcation point characterizing hotspot equilibria. For the time-dependent problem, a differential algebraic (DAE) system characterizing the slow dynamics of a collection of hotspots is derived, and the results compared favourably with full numerical simulations of the PDE system. The asymptotic theory to construct hotspot equilibria, and to derive the differential algebraic system

for quasi-steady patterns, is based on the resolution of a triple-deck structure near the core of each hotspot and the identification of so-called switchback terms.

[78] W.H. Tse and M.J. Ward. Hotspot formation and dynamics for a continuum model of urban crime. *European Journal of Applied Mathematics*, 27:583–624, 2015.

It is a well known fact that crime does not tend to be distributed evenly. Instead, crime is generally concentrated in certain locations more than others. These locations are sometimes referred to as crime hotspots. This article would be appropriate for anyone doing their research on crime hotspots and the distribution of crime. The authors use complex mathematics to model and predict the formation of crime hotspots. This article is widely cited, suggesting that it is a robust and credible resource.

[79] George Tsebelis. Penalty has no impact on crime. *SageJournals*, 1990.

This journal uses a game-theoretic framework to examine crime as a game between criminals and police. As a result, it is found that when the severity of the penatly has no impact on criminal behavior however does reduce the frequency of law enforcement.

[80] Yves van Gennip, Blake Hunter, Raymond Ahn, Peter Elliott, Kyle Luh, Megan Halvorson, Shannon Reid, Matthew Valasik, James Wo, George E. Tita, Andrea L. Bertozzi, , and P. Jeffrey Brantingham. Community detection using spectral clustering on sparse geosocial data. *SIAM Journal on Applied Mathematics*, 73:67–83, 2013.

The goal of this paper is to identify social communties among gang members in the Hollenbeck policing district. This is done by gathering information and creating a similarity graph for the individual gang members. Then spectral clustering is used to identify clusters in the graph which represent the communities in Hollenback and this is compared with the Los Angelos Police Department's information on gang membership.

[81] D. Wang, W. Ding, T Stepinski, J. Salazar, H. Lo, and M. Morabito. Optimization of criminal hotspots based on underlying controlling factors using geospatial discriminative pattern. In H. Jiang et al, editor, *Advanced Research in Applied Artificial Intelligence*, pages 553–562. Springer-Verlag Berlin Heidelberg, 2012.

This paper presents an algorithm called the Hotspots Optimization Tool that is used to improve crime hotspot identification by optimizing the boundaries of hotspots and generating a map. The method uses Geospatial Discriminative Pattern to build a

transaction-based geospatial dataset. The authors use Residential Burglary as the target crime in this study and consider not just the density of crime incidents, but also eight underlying socioeconomic controlling factors such as foreclosed homes and distance to local colleges. The algorithm also identifies areas in close proximity to hotspots, called footprints, that are at a higher risk of crime occurrence due to shared controlling factors. The authors include their code and clearly explain it line by line, as well as include an overview of related work and similar algorithms.

[82] Xiaofeng Wang, Donald E Brown, and Matthew S Gerber. Spatio-temporal modeling of criminal incidents using geographic, demographic, and twitter-derived information. *Intelligence and Security Informatics*, 2012.

The goal of this paper is to explain a new way to model the spatio-temporal pattern of criminal incidents. Two different methods were combined for this; the first being the spatio-temporal generalized additive model and the second being textual analysis.

[83] Wijayanto and Arie Wahyu. Fuzzy geographically weighted clustering using artificial bee colony: An efficient geodemographic analysis algorithm and applications to the analysis of crime behavior in population. *Applied Intelligence*, 44:377–398, 2016.

Geo-demographic analysis is an essential part of a geographical information system (GIS) for predicting people's behavior based on statistical models and their residential location. Fuzzy Geographically Weighted Clustering (FGWC) serves as one of the most efficient algorithms in geo-demographic analysis. Despite being an effective algorithm, FGWC is sensitive to initialize when the random selection of cluster centers makes the iterative process falling into the local optimal solution easily. Artificial Bee Colony (ABC), one of the most popular meta-heuristic algorithms, can be regarded as the tool to achieve global optimization solutions. This research aims to propose a novel geo-demographic analysis algorithm that integrates FGWC to the optimization scheme of ABC for improving geo-demographic clustering accuracy. Experimental results on various datasets show that the clustering quality of the proposed algorithm called FGWC-ABC is better than those of other relevant methods. The proposed algorithm is also applied to a decision-making application for analyzing crime behavior problem in the population using the US communities and crime dataset. It provides fuzzy rules to determine the violent crime rate in terms of linguistic labels from socioeconomic variables. These results are significant to make predictions of further US violent crime rate and to facilitate appropriate decisions on prevention such the situations in the future.

[84] Siu Kwong Wong. The effects of single-mother and single-father families on youth crime: Examining five gender-related hypotheses. *International Journal of Law, Crime and Justice*, 50:46–60, sep 2017.

This article examines Canadian single-parent families and tests several theories behind why youths from such families commit crime. It compares the concentration of single-mother and single-father families in an area to the youth crime rate in that area, and attempts to correlate that, using the theory as a guide, with an economic or social factor. For instance, one theory (which the article finds no evidence for) is that economically disadvantaged families are more likely to produce youth criminals, and since single-parent homes only have one income, they are especially susceptible. However, the article found that the effect of income in a region was weak compared to the effect of the concentration of single-parent families. Further results suggest that economic and financial factors are not the main cause of youth crime in single-parent families.

[85] J. Worral and T. Kovandzic. Police levels and crime rates: An instrumental variables approach. *Social Science Research*, 39:506–516, 2010.

This paper used panel data from over 5000 cities. The researchers estimated a series of fixed effects instrumental variable models using the Generalized Method of Moments to address the problems of endogeneity of police levels on crime rates. The conclusion is robust inverse associations between police levels and different crime rates.

[86] Dingqi Yang, Terence Heaney, Alberto Tonon, Leye Wang, and Philippe Cudré-Mauroux. Crimetelescope: crime hotspot prediction based on urban and social media data fusion. *World Wide Web*, pages 1–25, 2017.

In this article, the authors described CrimeTelescope, an online crime prediction and visualization platform based on urban and social media data fusion. The results showed that CrimeTelescope achieve accurate crime prediction via data fusion.This article is a good resource for future work.